Chapter 1

equation forms

general form: $Ax+By+C=0$ $\quad m=-\frac{A}{B}$ $\quad$ y-int $=(0,-\frac{C}{B})$

vertical line $\quad x=a$

horizontal line $\quad y=b$

slope-intercept form $\quad y=mx+b$

point-slope form $\quad y-y_1=m(x-x_1)$

2-point form $\quad y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_2)$

standard form $\quad ax+by=c$ $\quad m=-\frac{a}{b}$ $\quad$ y-int: $(0,\frac{c}{b})$

intercept form $\quad \frac{x}{a}+\frac{y}{b}=1$ $\quad (a,0),(0,b)$ → ints

function end behavior

$x^2 \quad -x^2 \quad x^3 \quad -x^3$

extra notes

- inverse function is flip over $y=x$
- if a function is one-to-one (passes vert + hor line test) its inverse is a function

$-\frac{+}{-}\sqrt{\quad}$

difference quotient

$\frac{f(x+h)-f(x)}{h}$ $h\neq 0$ or $\frac{f(x)-f(a)}{x-a}$, $x\neq a$

transformations

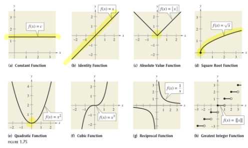
H: $y=f(x-c)$, $y=f(x-4)$, $y=f(x+3)$ → horizontal shift right/left
c > 0: "Horizontal shift right by 4 units"
c < 0: "Horizontal shift left by 3 units"

S: $y=f(x-c)$, $y=f(x-4)$, $y=f(x+3)$ → stretch/shrink - horizontal
b > 1: "horizontal stretch by a factor of" 3
0 < b < 1: "horizontal shrink bafo" $\frac{1}{2}$

$y=af(x)$, $y=2f(x)$, $y=\frac{1}{2}f(x)$ → stretch/shrink - vertical
a > 1: "vertical stretch bafo" 2
0 < a < 1: "vert shrink bafo" $\frac{1}{2}$

R: $y=-f(x)$ → Reflection across x-axis
$y=f(-x)$ → Reflection across y-axis

V: $y=f(x)+d$, $y=f(x)+5$, $y=f(x-3)$ → vertical shift up/down
d > 0: "vertical shift up by" 5 units
d < 0: "vertical shift down" by 3 units



even/odd

even → $f(-x)=f(x)$
$x\to -x$

odd → $f(-x)=-f(x)$
$x\to -x$
$y\to -y$

neither → neither even/odd

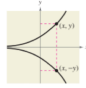
circles

std form: $(x-h)^2+(y-k)^2=r^2$

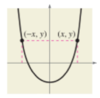
general: $ax^2+ay^2+dx+cy+f=0$
↳ use CTS

Symmetry

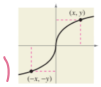
x-axis sym
→ $-f(x)=f(x)$



y-axis sym
→ even
→ $f(-x)=f(x)$



orgin sym
→ odd
→ $f(-x)=-f(x)$



Chapter 2

quadratic forms

definition form: ax^2+bx+c

standard form/vertex form: $f(x)=a(x-h)^2+k$ $\quad a\neq 0$

intercept form: $f(x)=a(x-h)(x-m)$

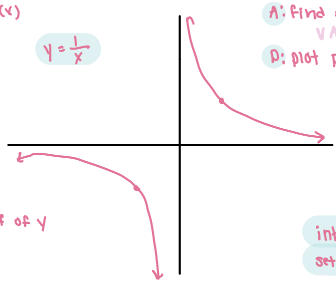
rational functions

$f(x)=\frac{p(x)}{q(x)}$ $\quad q(x)\neq 0$

Domain: all reals where $q(x)\neq 0$

Vert asym: factor of $q(x)$ that doesnt cancel out with $p(x)$
- if $(x-a)$ cancels) hole @ $x=a$

$y=\frac{1}{x}$



Hor asym: $\frac{x^n}{y^m}$

case 1: if $n<m$
- $y=0$ is HA

case 2: if $n=m$
- coef of x/coef of y

case 3: if $n>m$
- no HA

case 4: if n is 1 deg higher
- slant asymtote
- the quotient
- ignore remainder

graphing

multiplicity: repeated zero
- odd multiplicity, graph touches
- even multiplicity, graph touches (tangent)

turning points: if function with degree x_j max turning points $= x-1$

end behavior: if degree is odd, opposite directions
if leading coef is pos, right side goes up

graphing rational functions

(SIAP)

S: find symmetry

I: find intercepts
x-int (numerator = 0)
y-int (x = 0)

A: find Assymtotes
VA, HA, SA

P: plot points

descards rule of signs

- number of positive real zeros
→ equal/less then # of sign changes of f(x)
- number of negative real zeros
→ equal/less then # of sign changes of f(-x)

upper/lower bound

-3 | -1 3 -4 2 -8
- alterating
↳ -4 is lower bound

1 | 8 10 2 6 4
- all positive or 0
↳ x=1 is upper bound

inequalities

$x^2-2x-3>0$
$(x-3)(x+1)$, 3, -1

1. get into $f(x)<0$ or $f(x)>0$
2. solve $f(x)=0$ to get pts
3. create sign chart with pts
4. test values in each section for (+) or (-) result
5. write solution set

+ − + ; −1, 3

interval notation $(-\infty,-1)\cup(3,\infty)$

set-builder notation $x\mid x\leq -1$ or $x\geq 3$

rational inequalities

- same except step 2
↳ set num = 0
set denom = 0 } factors of num & denom

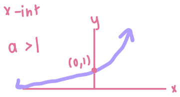
extra notes:

- standard form complex numbers $a+bi$
- complex zeros occur in pairs (+/-)

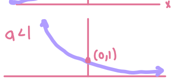
Chapter 3

exponential functions

Facts on $y=a^x$
- D: ℝ
R: $(0,\infty)$
- (0,1) always on the graph
- no x-int
- if $a>1$



- if $a<1$



- $y=a^x$ is one-to-one (inverse is a function)
- $y=0$ is HA

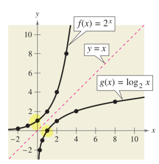
tranformations of $y=a^x$ $(c>0)$

H: $g(x)=a^{x+c}$ → c units left
$g(x)=a^{x-c}$ c-units right

S: hor: $g(x)=a^{cx}$ → $c>1$ shrink, $0<c<1$ stretch
vert: $g(x)=ca^x$ → $c>1$ stretch, $0<c<1$ shrink

R: $g(x)=-a^x$ → x-axis
$g(x)=a^{-x}$ → y-axis

V: $g(x)=a^x+c$ → up c units
$g(x)=a^x-c$ → down c-units



important formulas

compound interests

$A=P\left(1+\frac{r}{n}\right)^{nt}$; $n=$ # of compoundings per year

continuos interest

$A=Pe^{rt}$

half-life

$A=A_0\left(\frac{1}{2}\right)^{t/n}$

A = amt remaining
t = time
n = half-life
A_0 = initial Amt

solving examples

$\ln x-\ln 3=0$ $\qquad \ln x=\ln 3$ $\qquad x=3$

$e^x=7$ $\qquad \ln e^x=\ln 7$ $\qquad x=\ln 7$

$\ln x=-3$ $\qquad e^{\ln x}=e^{-3}$ $\qquad x=e^{-3}$

$\log x=-4$ $\qquad 10^{\log x}=10^{-4}$ $\qquad x=10^{-4}=\frac{1}{10}$

$\log_3 x=4$ $\qquad 3^{\log_3 x}=3^4$ $\qquad x=81$

logarithmic functions

$y = \log_a x \rightarrow x = a^y$

Change of base Formula

- $\log_a x = \frac{\log_b x}{\log_b a}$
- $\log_a x = \frac{\log x}{\log a}$
- $\log_a x = \frac{\ln x}{\ln a}$

properties of logs

logs

inverse prop
- $\log_a 1 = 0$ | $a^0 = 1$
- $\log_a a = 1$ | $a^1 = a$
- $\log_a a^x = x$ | $a^{\log_a x} = x$

one-to-one prop
- if $\log_a x = \log_a y$, $x = y$

natural logs
- $\ln 1 = 0$
- $\ln e = 1$
- $\ln e^x = x$, $e^{\ln x} = e$
- if $\ln x = \ln y$; $x = y$

product property

$\log_a(uv) = \log_a u + \log_a v$;
$\ln(uv) = \ln u + \ln v$

Quotient property

$\log_a \frac{u}{v} = \log_a u - \log_a v$

power property

$\log_a u^n = n \log_a u$

one-to-one

$a^x = a^y$ if $x = y$

$\log_a x = \log_a y$ if $x = y$

inverse

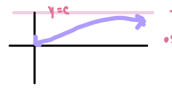
$a^{\log_a x} = x$

$\log_a a^x = x$

models

- exponential growth: $y = ae^{bx}$, $b > 0$
- exponential decay: $y = ae^{-bx}$, $b > 0$
- Gaussian model: $y = ae^{-(x-b)^2/c}$ → $y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$
  - normal distribution
  - bell-shaped
  - max val of function
    - ↳ average val of independent var

logistic growth model: $y = \frac{a}{1 + be^{-rx}}$



logarithmic: $y = a + b\ln x$, $y = a + b\log x$

$R = \log \frac{I}{I_0}$

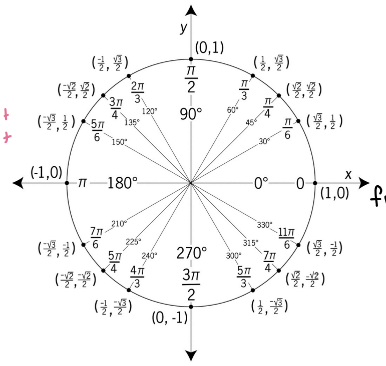
Chapter 4

even/odd functions

even:
- $\cos(-t) = \cos t$
- $\sec(-t) = \sec(t)$

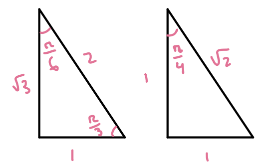
odd
- $\sin(-t) = -\sin t$
- $\tan(-t) = -\tan t$
- $\csc(-t) = -\csc t$
- $\cot(-t) = -\cot t$

unit circle



cofunction identities

- $\sin(90° - \theta) = \cos\theta$
- $\tan(90° - \theta) = \cot\theta$
- $\sec(90° - \theta) = \csc\theta$
- $\cos(90° - \theta) = \sin\theta$
- $\cot(90° - \theta) = \tan\theta$
- $\csc(90° - \theta) = \sec\theta$



period

2π { sin, cos, sec, csc }

tan, cot } π

fundamental trigonometric identities

reciprocal identities

- $\sin\theta = \frac{1}{\csc\theta}$
- $\cos\theta = \frac{1}{\sec\theta}$
- $\tan\theta = \frac{1}{\cot\theta}$
- $\csc\theta = \frac{1}{\sin\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$
- $\cot\theta = \frac{1}{\tan\theta}$

Quotient identities

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean identities

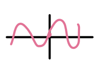
- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \csc^2\theta$

graphing sin/cos

$y = A\sin(bx - c) + D$ or $y = A\cos(Bx - c) + D$

- $|A|$ = amplitude
- $\frac{2\pi}{B}$ = period
- $\frac{c}{B}$ = hor shift
  - $\frac{c}{B}$ → starting pt
  - $\frac{c}{B} + \frac{2\pi}{B}$ → ending pt
- D = vertical shift
  - y=D ↳ center line

sin		cos	
start: $\frac{c}{B}$	D	start: $\frac{c}{B}$	max
qtr pt 1	max	qtr pt 1	D
half pt	D	half pt	min
qtr pt 3	min	qtr pt 3	D
$\frac{c}{B} + \frac{2\pi}{B}$	D	$\frac{c}{B} + \frac{2\pi}{B}$	max



- half pt → $\frac{\text{start} + \text{end}}{2}$
- qtr pt 1 → $\frac{\text{start} + \text{half}}{2}$
- qtr pt 3 → $\frac{\text{half} + \text{end}}{2}$

inverse functions

$\sin^{-1}(x)$ → I and IV
- D: $(-1 \le x \le 1)$
- R: $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\tan^{-1}(x)$ → I and IV
- D: $-\infty < x < \infty$
- R: $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\cos^{-1}(x)$ → (I and II)
- D: $-1 \le x \le 1$
- R: $0 \le \theta \le \pi$

sin, cos, tan
↳ Dom/range is switched

frequency & period

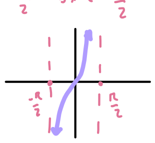
freq = $\frac{\omega}{2\pi}$

period = $\frac{2\pi}{\omega}$

graphing tan/cot

tan

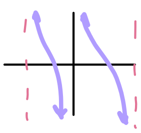
$-\frac{\pi}{2} < Bx - c < \frac{\pi}{2}$



- Qtr pt
  - ↳ -A to +A

cot

$0 < Bx - c < \pi$

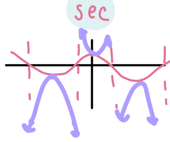


- Qtr pt
  - ↳ +A to -A

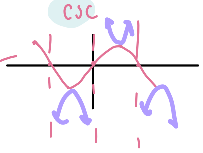
graphing sec/csc

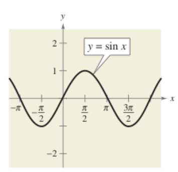
- graph cos/sin
- create asymptotes

sec

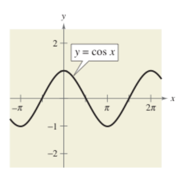


csc

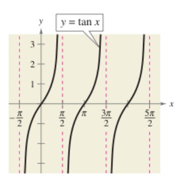




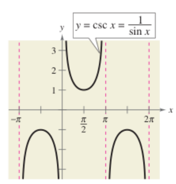
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π



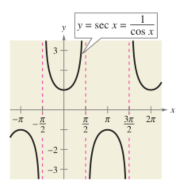
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π



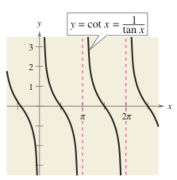
Domain: all $x \ne \frac{\pi}{2} + n\pi$
Range: $(-\infty, \infty)$
Period: π



Domain: all $x \ne n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π



Domain: all $x \ne \frac{\pi}{2} + n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π



Domain: all $x \ne n\pi$
Range: $(-\infty, \infty)$
Period: π

Figure 4.70

Deg, min, sec

$.36°$
- $.36 \times 60 = 21.6$
- $.6 \times 60 = 36$
- $0° \; 21' \; 36''$

$-408° \; 16' \; 20''$
- $-408 + \frac{16}{60} + \frac{20}{3600}$
- $-408.272°$

extra notes

- $\omega = \frac{\Delta\theta}{\Delta t}$

Chapter 5

fundamental trigonometric identities

reciprocal
- $\sin\theta = \frac{1}{\csc\theta}$ $\cos\theta = \frac{1}{\sec\theta}$ $\tan\theta = \frac{1}{\cot\theta}$
- $\csc\theta = \frac{1}{\sin\theta}$ $\sec\theta = \frac{1}{\cos\theta}$ $\cot\theta = \frac{1}{\tan\theta}$

Quotient
- $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean
- $\sin^2\theta + \cos^2\theta = 1$ $1+\tan^2\theta = \sec^2\theta$ $1+\cot^2\theta = \csc^2\theta$

cofunction
- $\sin(\frac{\pi}{2}-u) = \cos\theta$ $\cos(\frac{\pi}{2}-\theta) = \sin\theta$
- $\tan(\frac{\pi}{2}-u) = \cot\theta$ $\cot(\frac{\pi}{2}-\theta) = \tan\theta$
- $\sec(\frac{\pi}{2}-\theta) = \csc\theta$ $\csc(\frac{\pi}{2}-\theta) = \sec\theta$

even/odd
- $\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$
- $\csc\theta = -\csc\theta$ $\sec(-\theta) = \sec\theta$ $\cot(-\theta) = -\cot\theta$

sum and difference formulas

$\sin(u+v) = \sin u\cos v + \cos u \sin v$

$\sin(u-v) = \sin u\cos v - \cos u\sin v$

$\cos(u+v) = \cos u\cos v - \sin u \sin v$

$\cos(u-v) = \cos u\cos v + \sin u\sin v$

$\tan(u+v) = \frac{\tan u + \tan v}{1-\tan u\tan v}$ $\tan(u-v) = \frac{\tan u - \tan v}{1+\tan u\tan v}$

double-angle formulas

$\sin 2u = 2\sin u\cos u$

$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$

$\cos 2u = \cos^2 u - \sin^2 u$

$= 2\cos^2 u - 1$

$= 1 - 2\sin^2 u$

half-angle formulas

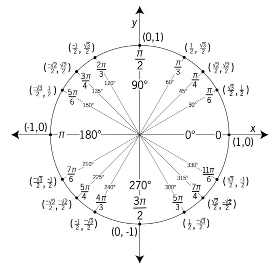
$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$

$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$

$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u}$

$= \frac{\sin u}{1+\cos u}$

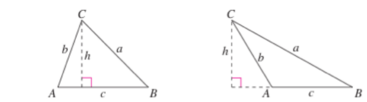
$= \pm\sqrt{\frac{1-\cos u}{1+\sin u}}$



Chapter 6

laws of sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



laws of cosines

$a^2 = b^2+c^2-2bc\cos A$ $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$b^2 = a^2+c^2-2ac\cos B$ $\cos B = \frac{a^2+c^2-b^2}{2ac}$

$c^2 = a^2+b^2-2ab\cos C$ $\cos C = \frac{a^2+b^2-c^2}{2ab}$

Area of oblique Δ

$A = \frac{1}{2}s_1 \cdot s_2 \cdot \sin(\text{incl} \angle)$

Heron's formula for area of a Δ

$A = \sqrt{s(s-a)(s-b)(s-c)}$ $s = \frac{a+b+c}{2}$

vector of 2 points

initial terminal

$P(p_1, p_2)$ $Q(q_1, q_2)$

$\vec{PQ} = \langle q_1-p_1, q_2-p_2\rangle = \langle v_1, v_2\rangle = v$

magnitude

$\|v\| = \sqrt{v_1^2+v_2^2}$

unit vector

$u = \frac{v}{\|v\|} = \left(\frac{1}{\|v\|}\right)v$

direction angles

$\tan\theta = \frac{b}{a}$

num to trig

$v = \|v\|\cos\theta\, i + \|v\|\sin\theta\, j$

dot product

$u \cdot v = u_1v_1 + u_2v_2$

- vectors $u \cdot v$ are orthagonal if $u\cdot v = 0$
- vectors $u \cdot v$ are perpendicular if $u \cdot v = -1$
- vectors $u \cdot v$ are neither if $u \cdot v = 1$

angle between two vectors

$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$

projection of u onto v

$\text{proj}_v u = \left(\frac{u\cdot v}{\|v\|^2}\right)v$

$u = w_1 + w_2$

$w_1 = \text{proj}_v u$

work

$W = Fd\cos\theta$

$W = \|\text{proj}_{PQ} F\|\|\vec{PQ}\|$

$W = F \cdot \vec{PQ}$

product/quotient of complex num

$z_1 = r_1(\cos\theta_1 + i\sin\theta)$ $z_2 = (\cos\theta_2 + i\sin\theta_2)$

$z_1 z_2 = r_1 r_2 [\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)], \; z_2 \neq 0$

DeMoivre's Theorem

$z = r(\cos\theta + i\sin\theta)$

$z^n = [r(\cos\theta + i\sin\theta)]^n$

$= r^n(\cos n\theta + i\sin n\theta)$

n^{th} roots n → constant

$\sqrt[n]{r}\left(\cos\left(\frac{\theta+2\pi k}{n}\right) + i\sin\left(\frac{\theta+2\pi k}{n}\right)\right)$

$n = 0, 1, 2, \dots, n-1$

Chapter 7

on gc
menue
↳ algebra
↳ expand

partial fraction 4 cases

distict linear factors

$\frac{5x-1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$

repeated linear & prime quadratic

$\frac{10x^2+2x}{(x-1)^2(x^2+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+d}{x^2+2}$

repeated linear factors

$\frac{3x^2+49}{x(x+7)^2} = \frac{A}{x} + \frac{B}{x+7} + \frac{C}{(x+7)^2}$

repeated prime quadratic

$\frac{3x^3-6x^2+7x-2}{(x^2-2x+2)^2} = \frac{Ax+B}{x^2-2x+2} + \frac{Cx+D}{(x^2-2x+2)^2}$

Chapter 8

inverse

option 1

$$\begin{bmatrix}1 & 4\\-1 & -3\end{bmatrix}\begin{bmatrix}x_{11} & x_{12}\\x_{21} & x_{22}\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

A x I

$$\begin{bmatrix}(x_{11}+4x_{21}) & (x_{12}+4x_{22})\\(-x_{11}-3x_{21}) & (-x_{12}-3x_{22})\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

option 2

$$\left[\begin{array}{cc|cc}1 & 4 & 1 & 0\\-1 & -3 & 0 & 1\end{array}\right] \; (A \vdots I)$$

$$\left[\begin{array}{cc|cc}1 & 4 & 1 & 0\\0 & 1 & 1 & 1\end{array}\right] \rightarrow R_1 + R_2$$

$$\left[\begin{array}{cc|cc}1 & 0 & -3 & -4\\0 & 1 & 1 & 1\end{array}\right] \rightarrow -4R_2 +$$

$A^{-1} = \begin{bmatrix}-3 & -4\\1 & 1\end{bmatrix}$

option 3

$A = \begin{bmatrix}a & b\\c & d\end{bmatrix} \longrightarrow A^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$

Systems of equations

$$[A]^{-1} \times [\text{solution}] = \begin{bmatrix}x\\y\\z\end{bmatrix}$$

reduced row-echelon form

$$\begin{bmatrix}1 & 0 & 4\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

row-echalon form

$$\begin{bmatrix}1 & 3 & 4\\0 & 1 & 8\\0 & 0 & 1\end{bmatrix}$$

matrix multiplication

$A = \begin{bmatrix}-1 & 3\\4 & 2\\5 & 0\end{bmatrix}$ $B = \begin{bmatrix}2 & 5 & 1 & -2\\4 & -3 & 0 & -1\end{bmatrix}$

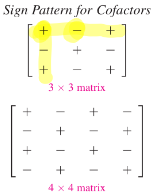
$(1,1)$
$-1\cdot 2 + 3\cdot 4$
10

$(3,2)$
$25+0$
25

$AB = \begin{bmatrix}10 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 25 & 0 & 0\end{bmatrix}$

Determinate

Sign Pattern for Cofactors



$\begin{bmatrix}a_1 & b_1\\a_2 & b_2\end{bmatrix}$

$|A| = a_1b_2 - a_2b_1$

$\begin{bmatrix}0 & 2 & 1\\3 & -1 & 2\\4 & 0 & 1\end{bmatrix}$ $|A| = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

$0 - 2\begin{vmatrix}3 & 2\\4 & 1\end{vmatrix} + 1\begin{vmatrix}3 & -1\\4 & 0\end{vmatrix}$

$|A| = 14$

cramer's rule when $D \neq 0$

Area of a triangle

collinear points

equation of a line

$4x - 2y = 10$
$3x - 5y = 11$

$x = \frac{D_x}{D}$ $y = \frac{D_y}{D}$

$D_x = \begin{vmatrix} 10 & -2 \\ 11 & -5 \end{vmatrix}$ $D_y = \begin{vmatrix} 4 & 10 \\ 3 & 11 \end{vmatrix}$

$(x_1, y_1)\ (x_2, y_2)\ (x_3, y_3)$

$\text{Area} = \pm \frac{1}{2} \begin{vmatrix} x_1 & x_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

if $\begin{vmatrix} x_1 & x_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

$\begin{vmatrix} x & y & 1 \\ 2 & 4 & 1 \\ -2 & 3 & 1 \end{vmatrix}$ $x\begin{vmatrix} 4 & 1 \\ 3 & 1 \end{vmatrix} - y\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 4 \\ -1 & 3 \end{vmatrix}$

$x - 3y + 10 = 0$

Cryptography

coding: $P \cdot A = C$ (original → P, coded → C)

decoding: $C \times A^{-1} = P$

Chapter 9

Arithmatic

$a_n = a_{n-1} + d$ $a_n = a_1 + (n-1)d$

sum of first n terms

$S_n = \frac{n}{2}(a_1 + a_n)$ $\sum_{n=1}^{?} a_1 + (n-1)d$

note: $a_{10} = a_4 r^{10-4}$

compound interest: $A = \text{initial}\,(1 + \text{rate})^{\text{time}}$

Geometric

$a_n = a_{n-1} * r$ $a_n = a_1 \times r^{n-1}$

sum of first n terms

$S_n = \frac{a_1(1 - r^n)}{(1-r)}$ $\sum_{n=1}^{?} a_1 \times (r)^{n-1}$

sum of infinite series: $S = \frac{a_1}{(1-r)}$

Binomials

$(x-y)^{10}$

$\binom{10}{0} x^{10} y^0 + \binom{10}{1} x^9 (-y)^1 + \binom{10}{2} x^8 (-y)^2 \ldots$

r^{th} term of $(a+b)^n$: $\binom{n}{r-1} a^{n-r+1} b^{r-1}$

permutations ABC != BCA

- how many permutations for ABCD
  $\underline{4} \cdot \underline{3} \cdot \underline{2} \cdot 1 = 4!$
- 8 horses, how many ways can 3 finish?
  $\underline{8} \cdot \underline{7} \cdot \underline{6} = \left(\frac{8!}{(8-3)!}\right) = {}_8P_3$
- how many ways rearange ABBCCD?
  $\frac{n!}{n_1! \cdot n_2! \cdot n_3!}$ $\frac{6!}{1!2!2!1!}$

$n = n_1 + n_2 + n_3$

Combinations ABC = CBA

n = # elements

r = # spots

${}_nC_r = \frac{n!}{(n-r)!\,r!}$

Probability

$\frac{\text{favorable outcomes}}{\text{possible outcomes}}$

probability of A or B

$\underbrace{P(A \cup B)}_{\text{A or B}} = P(A) + P(B) - \underbrace{P(A \cap B)}_{\text{probability of both}}$

probability of A and B

$P(A \text{ and } B) = P(A) \cdot P(B)$

probability of a complement

$P(A') = 1 - P(A)$

Chapter 10

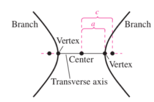
$m = \tan\theta$ (slope ← m, inclination ← θ)

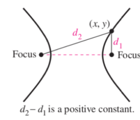
angle b/w 2 lines

$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$

distance b/w pt. and line $Ax + By + C = 0$

$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$





$d_2 - d_1$ is a positive constant.

Parabola

	$(x-h)^2 = 4p(y-k)$	$(y-k)^2 = 4p(x-h)$
Directrix	$y = k - p$	$x = h - p$
focus	$(h, k+p)$	$(h+p, k)$
	$p > 0$ opens up; $p < 0$ opens down	$p > 0$ opens right; $p < 0$ opens left

latus rectum = $4p$

Ellipse

$c^2 = a^2 - b^2,\ 0 < b < a$

	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} - \frac{(y-k)^2}{a^2} = 1$
Major axis:	horizontal w/length $2a$	vertical w/length $2a$
minor axis:	vertical w/length $2b$	horizontal w/length $2b$
vertices [endpts of MA]:	$(h \pm a, k)$	$(h, k \pm a)$
foci:	$(h \pm c, k)$	$(h, k \pm c)$
endpts of minor axis:	$(h, k \pm b)$	$(h \pm b, k)$

eccentricity $e = \frac{c}{a}$

Hyperbola

$c^2 = a^2 + b^2$

	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
transverse axis:	horizontal → $2a$	vertical $2a$
conjugate axis:	vertical → $2b$	horizontal $2b$
vertices (endpts of trans axis):	$(h \pm a, k)$	$(h, k \pm a)$
foci:	$(h \pm c, k)$	$(h, k \pm c)$
asymtotes:	$y - h = \pm \frac{b}{a}(x - h)$; $y = h \pm \frac{b}{a}(x-h)$	$y - h = \pm \frac{a}{b}(x - h)$; $y = h \pm \frac{a}{b}(x - h)$

$e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a}$

$|d_2 - d_1| = 2a$

parametric equations ★ use →

t	x	y

- graph (x, y)

eliminate t
- solve for t
- plug in

eliminate angle
- simplify $\cos\theta$ / $\sin\theta$
- use identities

find parametric equation
- plug in known t values

ellipse

$x = h + b\cos\theta$
$y = k + a\sin\theta$

Polar Quordinates

on gc: menu, 5num, ↳ complex num tools

polar → cart
- $x = r\cos\theta$
- $y = r\sin\theta$

cart → polar
- $\tan\theta = \frac{y}{x}$
- $r^2 = x^2 + y^2$

★ when r left, $r = \sqrt{x^2 + y^2}$

test symmetry: replace (r, θ)

line $\theta = \pi/2$ → $(r, \pi - \theta)$ or $(-r, -\theta)$

polar axis → $(r, -\theta)$ or $(-r, \pi - \theta)$

the pole → $(r, \pi + \theta)$ or $(-r, \theta)$

$Ax^2 + Cy^2 + Dx + Ey + F = 0$

circle: $A = C$

parabola: $AC = 0$

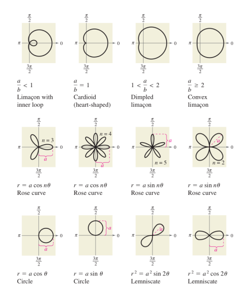
ellipse: $AC > 0$

hyperbola: $AC < 0$

Limaçons
$r = a \pm b\cos\theta$
$r = a \pm b\sin\theta$
$(a > 0, b > 0)$

Rose Curves
n petals if n is odd,
$2n$ petals if n is even
$(n \geq 2)$.

Circles and Lemniscates



Chapter 12

$\lim_{x \to c}$ Methods

method 1 → plug in algebriacly

method 2 → rationilize + cancel out

method 3 → use gc

$\lim_{x \to \infty}$ Methods

if deg num = deg den → simplify to highest power and use coefficients

if deg num < deg den → limit = 0

if deg num > deg den → either $-\infty$ or ∞

Properties $\left(\lim_{x \to c} f(x) = L\right) \left(\lim_{x \to c} g(x) = K\right)$

- scalar multiple: $\lim_{x \to c} [b f(x)] = bL$
- sum/difference: $\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
- product: $\lim_{x \to c} [f(x)\, g(x)] = LK$
- Quotient: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}$
- power: $\lim_{x \to c} [f(x)]^n = L^n$

Sequences

- which value does the sequence approach
- if it diverges → doesn't approach anything